Electromagnetic

Lecture (5)

Chapter (2)

"Gauss Law"

2nd year

communications

مركز النور
للخدمات العلمية
٠١٠٠٩٣٦٤٠٠٩١

مركز النور
للخدمات العلمية
٠١٠٠٩٣٦٤٠٠٩١

Lecture (5):

Chapter (2)

Gauss Law

1 Electric Flux: (Ψ)

⇐ هو عبارة عن الفيض الكهربى الذى يخرج من أى شحنة

⇐ هذا الفيض الكهربى يتناسب طردياً مع قيمة الشحنة ذاتها.

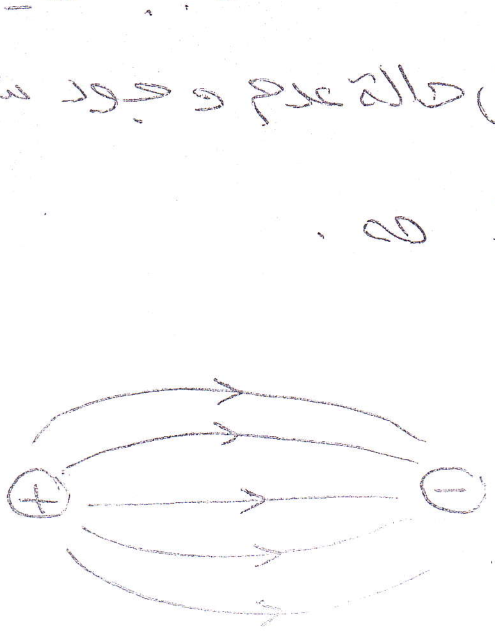
$$\Psi \propto Q$$

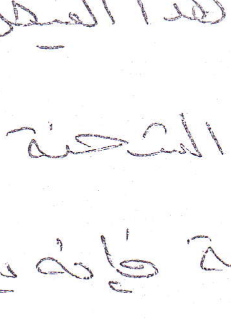
$$\frac{h\,du_1}{Eu_1} = \frac{h\,du_2}{Eu_2}$$

وقد وُجد أن ثابت التناسب = 1

$$\therefore \boxed{\Psi = Q}$$

⇐ وهذا الفيض يخرج من الشحنة الموجبة وينتهى عند الشحنة السالبة وفى حالة عدم وجود شحنة سالبة فإنه ينتهى عند الـ ∞.





[2] Electric Flux Density: $\bar{D}$

⇐ **كثافة الفيض الكهربي** : هى عدد خطوط الفيض الكهربي التى تمر عمودياً على وحدة المساحات .

$$\boxed{\psi = \int_S \bar{D} \cdot d\bar{S}}$$

ψ

$\psi = \int_s \bar{D} \cdot d\bar{S}$

⇒ **Note :** $\boxed{\bar{D} = \epsilon \bar{E}}$

∴ يمكن حساب الـ $\bar{D}$ عن طريق حساب الـ $\bar{E}$ أو العكس .

Sheet (3)

[1] $Q = 30\ nC \rightarrow (0,0,0)$

Find $\bar{D}$ at $P(1,3,-4)$

$\boxed{\text{Solution}}$

$\bar{R} = (1,3,-4) - (0,0,0) = \hat{x} + 3\hat{y} - 4\hat{z}$

$|R| = \sqrt{1+9+16} = \sqrt{26}$

$\therefore \bar{E} = \frac{Q}{\quad} \cdot \frac{\bar{R}}{|R|^3}$

$$\therefore \bar{E} = \frac{Q}{4\pi\epsilon} \cdot \frac{\hat{x} + 3\hat{y} - 4\hat{z}}{(26)^{3/2}}$$

$$\bar{D} = \epsilon \bar{E} = \frac{Q}{4\pi} \cdot \frac{\hat{x} + 3\hat{y} - 4\hat{z}}{(26)^{3/2}}$$

$$= 0.018(\hat{x} + 3\hat{y} - 4\hat{z}) \ nC/m^2 .$$

$\boxed{3}$ Gauss Law :

" The total Flux out of a Closed sur. is equal to the net charge within the sur. "

" الفيض الكلي الخارج من أي سطح مغلق يساوي الشحنة الكلية المحصورة داخل هذا السطح المغلق ".

$$\boxed{\Psi = \oint_S \bar{D} \cdot d\bar{S} = Q_{enc}}$$ حفظ

Ψ ≡ Total Flux out of the Closed sur.

Q_{enc} ≡ The net enclosed charge in sur.

$\oint_S$ ≡ Integration over a Closed sur. (S).

$\cdot$ ≡ dot product .

• لحساب الـ $\bar{D}$ لأى نوع من الشحنات باستخدام قانون جاوس لابد أولاً أن نفرض سطح مغلق مناسب لنوع الشحنة وهذا السطح يسمى سطح جاوس.

⇛ Gaussian Sur. Conditions:

(1) This sur. is totally enClosed.

(2) Symmetric about the Charge.

Charge	Gaussian Sur
Line	Cyl.
Cyl	Cyl
Point	Sph.
Sph.	Sph

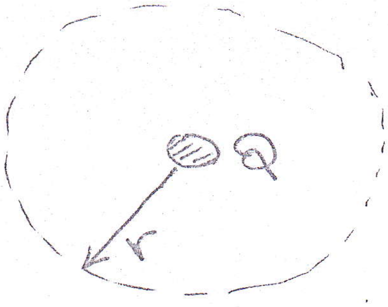
x.2 : find $\bar{E}$ at any point due to a point charge at the origin, using Gauss law.

Solution

Charge : point charge.

gaussian sur. : sphere

Gauss law:

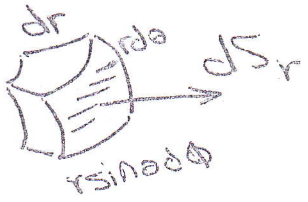


$$\oint_s \bar{D} \cdot d\bar{S} = Q_{enc}$$

$D_r \; \hat{r}$

$\bar{D} = D_r \; \hat{r} \dashrightarrow$ (لأن المركبة العمودية لـ $\bar{D}$ على سطح جاوس هي $\hat{r}$ فقط)

$dS_r = r^2 \sin\theta \, d$



$$d\bar{S} = dS_r \, \hat{r} = r^2 \sin\theta \, d\theta \, d\phi \; \hat{r}$$

$$\therefore \bar{D} \cdot d\bar{S} = D_r \, r^2 \sin\theta \, d\theta \, d\phi$$

$Q_{enc} = 4\pi r^2 D_r$

$D_r = Q / 4\pi r^2$

$$Q_{enc} = Q$$

$$\therefore Q = \int_0^{2\pi} \int_0^{\pi} D_r \, r^2 \sin\theta \, d\theta \, d\phi$$

$$Q = D_r \cdot r^2 \cdot \cos\theta \Big|_{\pi}^{0} \cdot 2\pi$$

$$Q = 4\pi r^2 D_r$$

$$\therefore \bar{D} = \frac{Q}{4\pi r^2}\,\hat{r}$$

$$\boxed{\bar{E} = \frac{\bar{D}}{\epsilon} = \frac{Q}{4\pi\epsilon r^2}\,\hat{r}}$$

⇒ Note :

شرط إختيار سطح جاوس بأنه متماثل حول الشحنة أدى إلى
أن قيمة الـ $\bar{D}$ عند هذا السطح تكون قيمة ثابتة فبالتالي
يمكن أن تخرج كثابت خارج التكامل.

$$\oint \bar{D}\cdot d\bar{S} \Rightarrow \bar{D}\cdot\bar{S}$$

مركز الفجر
للخدمات العلمية
٠١٠٠٩٢٦٤٠٠٦

. for the pervious example :

$$\bar{D} = D_r\,\hat{r}$$

$$\bar{S} = \underbrace{4\pi r^2}\,\hat{r}$$

مساحة سطح الكرة (سطح جاوس)
بدون تكامل لأن D_r ثابتة
على هذا السطح.

$$\therefore Q = \bar{D}\cdot\bar{S}$$

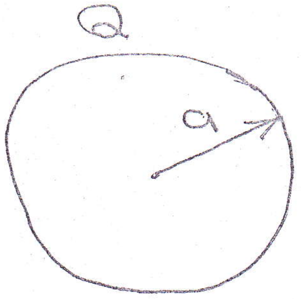
$$= 4\pi r^2 D_r$$

$$D_r = \frac{Q}{4\pi r^2} \qquad \therefore \bar{D} = \frac{Q}{4\pi r^2}\,\hat{r}$$

ex.3 : find $\bar{E}$ of a hollow sphere with a charge Q, using gauss law.

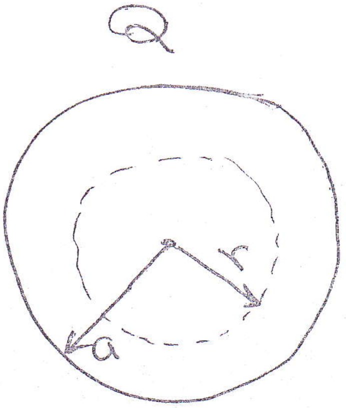
Solution

- hollow sph : كرة مفرغة من الداخل



- Gaussian sur : sphere with radius r

وبالنسبة للأجسام نقوم بحساب الـ $\bar{D}$ في جميع المناطق



(a) $r < a$:

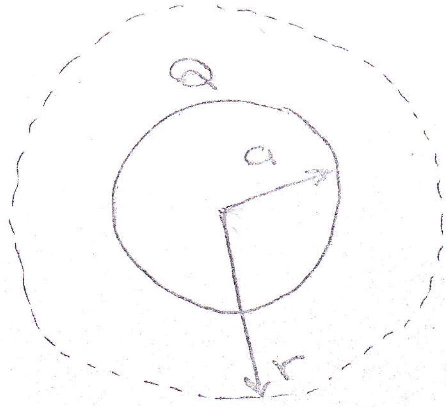
(سطح جاوس هو كرة نصف قطرها r
أقل من a)

$\therefore Q_{enc} = 0$ (لأن الكرة لا يوجد داخلها شحنة)

$$\bar{D} \cdot \bar{S} = Q_{enc} = 0$$

$$\therefore \bar{D} = 0$$

(b) $\underline{r > a}$:



$$\bar{D} \cdot \bar{S} = Q_{enc}$$

$$D_r \hat{r} \cdot 4\pi r^2 \hat{r} = Q$$

$$\therefore 4\pi r^2 D_r = Q$$

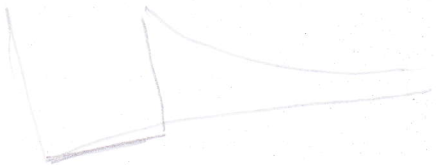
$$\therefore D_r = \frac{Q}{4\pi r^2} \qquad \bar{D} = \frac{Q}{4\pi r^2}\hat{r}$$

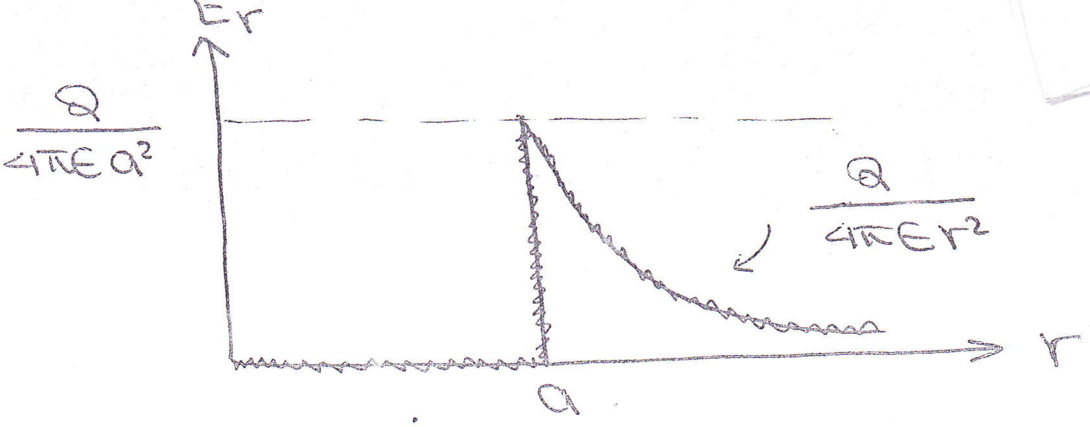
$$\therefore \bar{D} = \begin{cases} 0 & r < a \\ \dfrac{Q}{4\pi r^2}\hat{r} & r > a \end{cases}$$

D Q

$D_r \hat{r} \cdot 4\pi r^2 r$

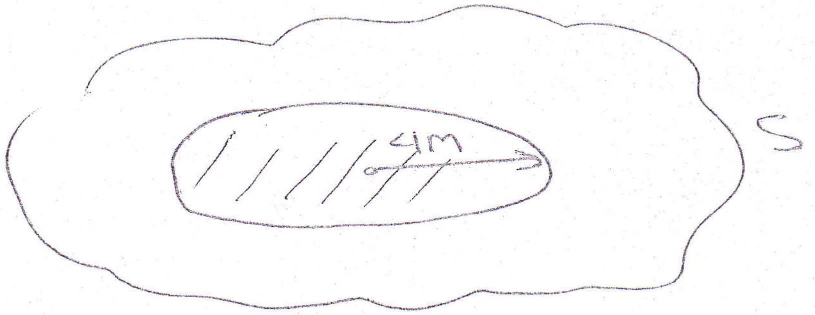
$$\bar{E} = \frac{\bar{D}}{\epsilon} = \begin{cases} 0 & r < a \\ \dfrac{Q}{4\pi\epsilon r^2}\hat{r} & r > a \end{cases}$$





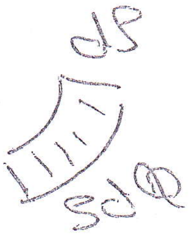
$\Rightarrow$ Sheet (3):

$\boxed{2}$ Cyl. disk of radius 4 m



$$\rho_s = \frac{\sin^2 \phi}{2\rho} \ C/m^2$$

$\Rightarrow$ Gauss law: $\Psi = Q_{enc}$



$$Q_{enc} = \int \rho_s \, ds$$

$$= \int_0^{2\pi} \int_0^4 \frac{\sin^2 \phi}{2\cancel{\rho}} \cancel{\rho} \, d\rho \, d\phi$$

$$= \frac{1}{2} \cdot \rho \Big|_0^4 \cdot \frac{1}{2} \int_0^{2\pi} 1 - \cos 2\phi \, d\phi$$

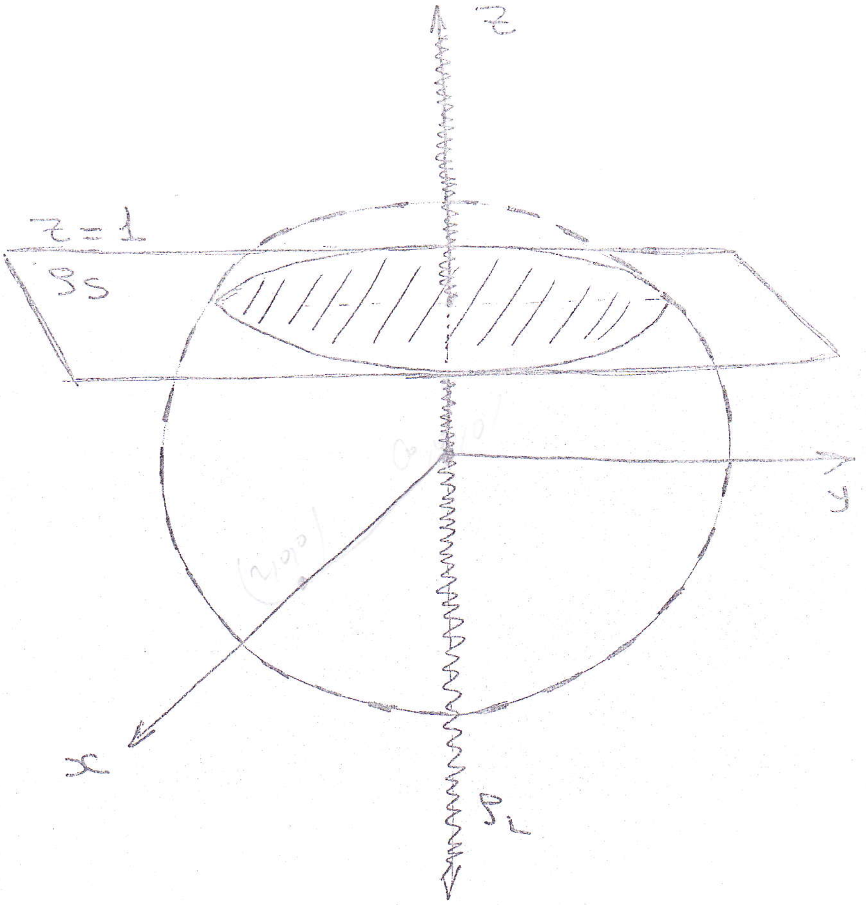
$$= \left[\phi - \frac{\sin 2\phi}{2} \right]_0^{2\pi} = 2\pi$$

$$\therefore \Psi = Q_{enc} = 2\pi \ C$$

$\boxed{3}$

$\rho_L = 15\ nC/m$

$\rho_S = 4\ nC/m^2$



(a)

Gauss law :

$Q_{enc} = \Psi$

- $Q_{enc} = Q_l + Q_S$

$Q_L = \rho_L \cdot l = 15 * 10^{-9} * 4 = 60\ nC$

$Q_S = \rho_S \cdot S \Leftarrow$ تقاطع المستوى مع الكرة يعطي دائرة

Sphere eqn : $x^2 + y^2 + z^2 = r^2$

$x^2 + y^2 + z^2 = 4$

plane sur. eqn : $z = 1$

$\rightarrow$ بحل المعادلتين يمكن الحصول على منطقة التقاطع (الدائرة)

- intersection region : $x^2 + y^2 + 1 = 4$

$x^2 + y^2 = 3$

$\therefore S = \pi r^2 = 3\pi$

$Q_S = \rho_S . S = 4 * 10^{-9} * 3\pi = 12\pi \; nC$

$\therefore Q_{enc} = Q_l + Q_S = (60 + 12\pi) \; nC$

note: we use $Q_L = \rho_L \, l$ and $Q_S = \rho_S \, S$

because ρ_L and ρ_S are const.

(b) sph eqn: $x^2 + y^2 + z^2 = 4$

Sur.: $x = 2$

$\therefore 4 + y^2 + z^2 = 4$

$$\underbrace{y^2 + z^2 = 0}$$

It is a circle eqn but its radius = 0 (point)

$\therefore$ It is a point $(2,0,0)$

$\therefore$ we need to find $|D|$ at point $(2,0,0)$

$\bar{E}_p = \bar{E}_l + \bar{E}_s$

$$\bar{l} = (2,0,0) - (0,0,0) = 2\hat{x}$$

$$|l| = 2$$

$$\bar{E}_l = \frac{\rho_L}{2\pi\epsilon} \cdot \frac{2\hat{x}}{4} = \frac{\rho_L}{4\pi\epsilon}\hat{x}$$

$$\bar{E}_s = \frac{\rho_s}{2\epsilon}\hat{n} = -\frac{\rho_s}{2\epsilon}\hat{z} \qquad (\hat{n} = -\hat{z})$$

$$\therefore \bar{E}_p = \frac{\rho_L}{4\pi\epsilon}\hat{x} - \frac{\rho_s}{2\epsilon}\hat{z}$$

$$\bar{D} = \epsilon\bar{E}_p = \frac{\rho_L}{4\pi}\hat{x} - \frac{\rho_s}{2}\hat{z}$$

$$\therefore |D| = \sqrt{\left(\frac{\rho_L}{4\pi}\right)^2 + \left(\frac{\rho_s}{2}\right)^2} = 2.33 \text{ nc/m}^2$$

$$\boxed{|D| = 2.33 \text{ nc/m}^2}$$

مركز الفجر
للخدمات الهندسية
٠١٠٠٩٣٦٤٠٠٦

$\boxed{4]}$

ⓐ

$$\rho_s = \begin{cases} 8 \quad nc/m^2 & r = 3m \\ -12 \quad nc/m^2 & r = 4m \\ \rho_s \quad nc/m^2 & r = 5m. \end{cases}$$

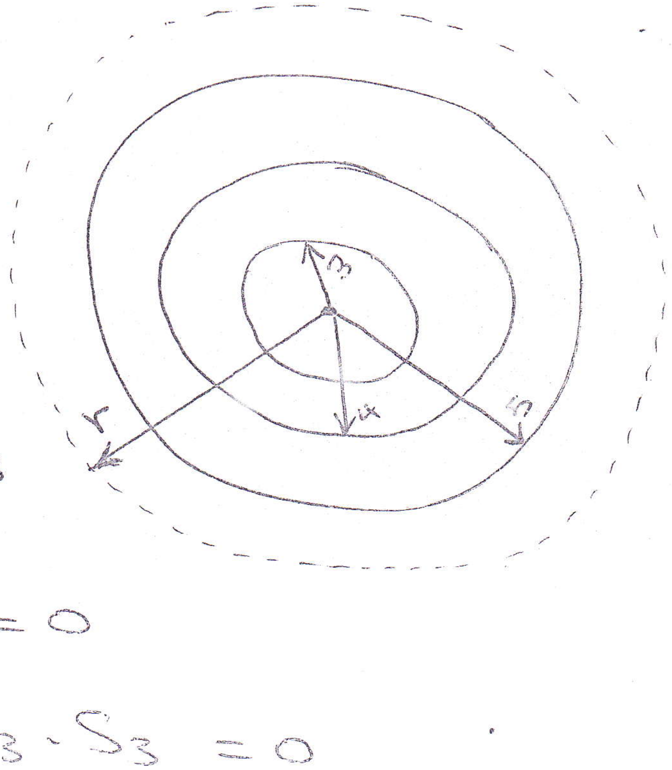
- find ρ_s so that $D=0$ for $r>5$.

$\boxed{\text{Solution}}$

- using gauss law :

gaussian sur. : sph with radius r

and $r>5$



We need $\bar{D} = 0$

$\therefore Q_{enc} = \oint_S \bar{D} \cdot d\bar{S} = 0$

but $Q_{enc} = Q_{s1} + Q_{s2} + Q_{s3}$

$\therefore Q_{s1} + Q_{s2} + Q_{s3} = 0$

$\rho_{s1} \cdot S_1 + \rho_{s2} \cdot S_2 + \rho_{s3} \cdot S_3 = 0$

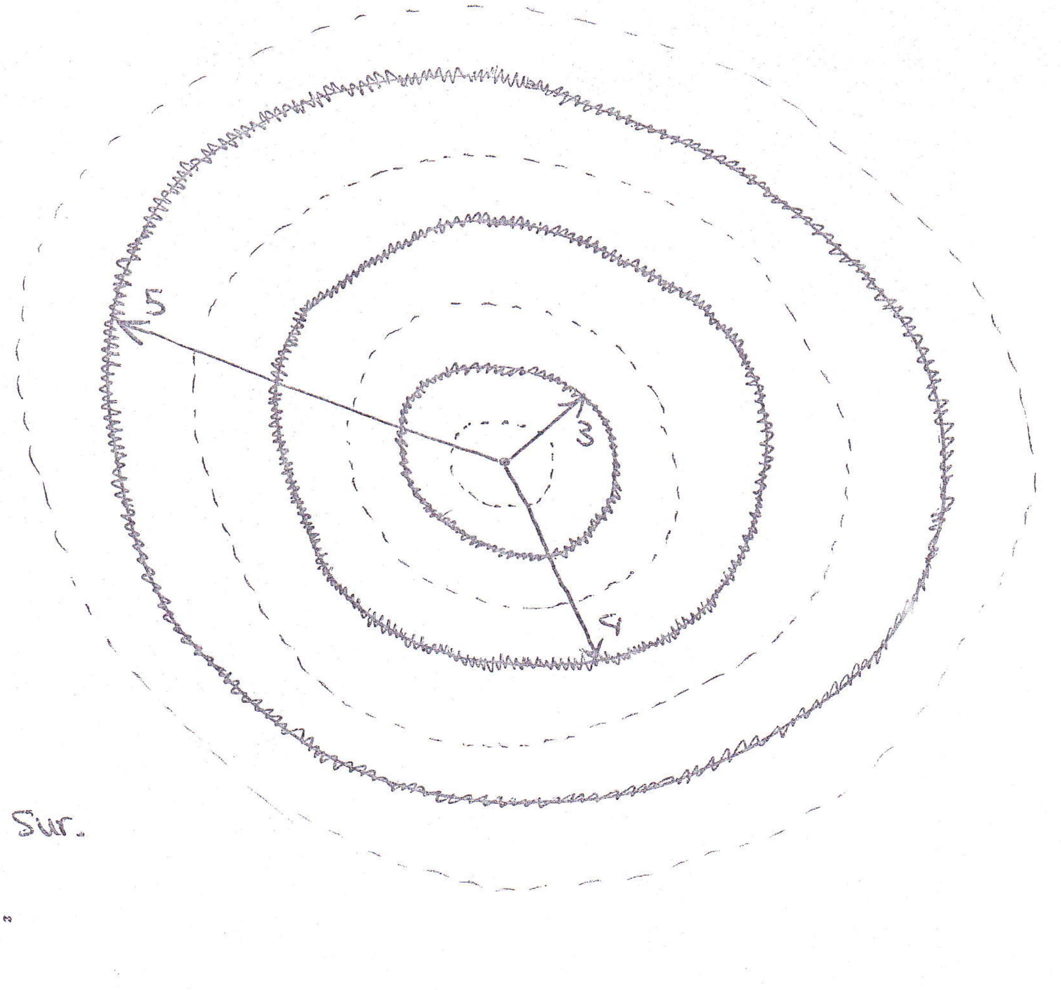
$8(4\pi \cdot 3^2) - 12(4\pi \cdot 4^2) + \rho_s(4\pi \cdot 5^2) = 0$

$\therefore \rho_s = 4.8 \; nC/m^2$

(b) plot D_r vs. r $(0<r<6)$.

$\therefore$ we need to Calculate $\bar{D}$ in all regions.

Gaussian surfaces : spheres with radius r



---- Gaussian sur.

~~~~ spheres.

(a) $\underline{r < 3}$ :

$Q_{enc} = 0$ (no charge)

$\therefore \bar{D} = 0$
~~~~

(b) $3 < r < 4$:

$$Q_{enc} = Q_{S1} = \rho_{S1} \cdot S_1 = 8(4\pi \cdot 3^2)$$

$$\bar{D} \cdot \bar{S} = 4\pi r^2 D_r$$

$$\therefore \cancel{4\pi} r^2 D_r = 8(\cancel{4\pi} \cdot 3^2)$$

$$D_r = \frac{72}{r^2} \text{ nC/m}^2$$

(c) $4 < r < 5$:

$$Q_{enc} = Q_{S1} + Q_{S2} = \rho_{S1} \cdot S_1 + \rho_{S2} \cdot S_2$$

$$= 8(4\pi \cdot 3^2) - 12(4\pi \cdot 4^2)$$

$$\bar{D} \cdot \bar{S} = 4\pi r^2 D_r$$

$$\therefore \cancel{4\pi} r^2 D_r = 8(\cancel{4\pi} \cdot 3^2) - 12(\cancel{4\pi} \cdot 4^2)$$

$$\therefore D_r = -\frac{120}{r^2} \text{ nC/m}^2$$

(d) $r > 5$:

$$Q_{enc} = Q_{S1} + Q_{S2} + Q_{S3}$$

$$\therefore 8(\cancel{4\pi} \cdot 3^2) - 12(\cancel{4\pi} \cdot 4^2) + 2(\cancel{4\pi} \cdot 5^2) = \cancel{4\pi} r^2 D_r$$

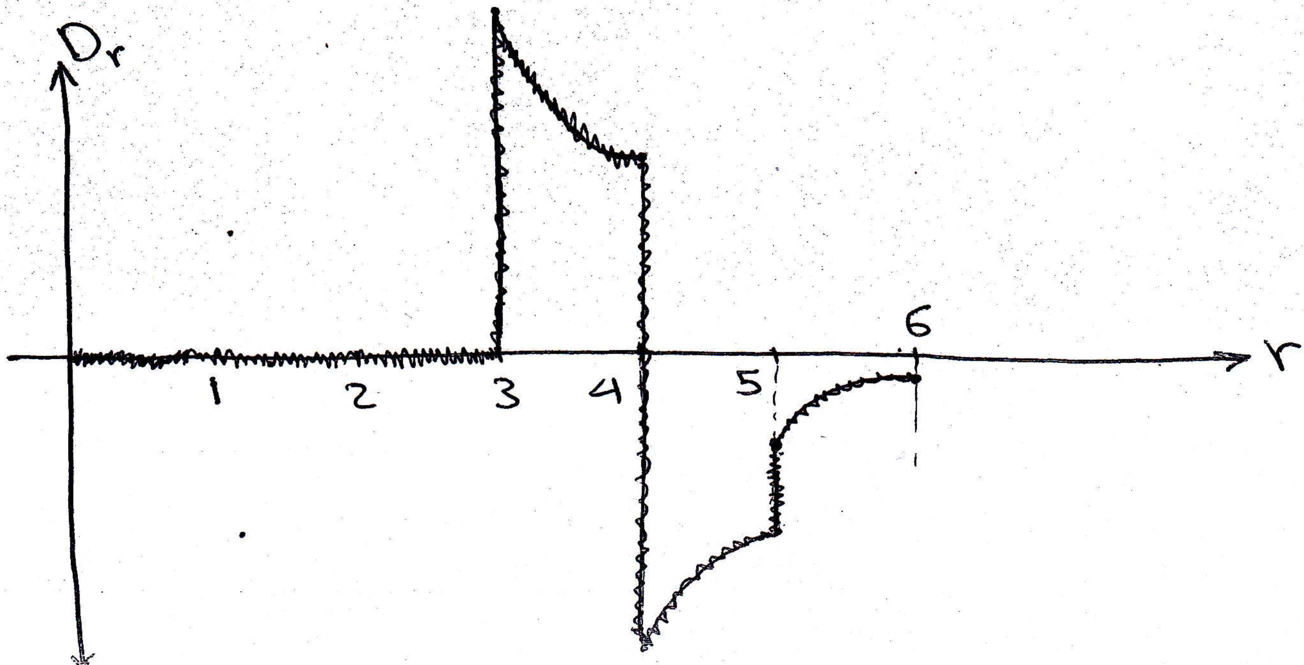
$$\therefore D_r = -\frac{70}{r^2} \text{ nc/m}^2$$

$$\therefore D_r = \begin{cases} 0 \longrightarrow & r < 3 \\ \frac{72}{r^2} \longrightarrow & 3 < r < 4 \\ -\frac{120}{r^2} \longrightarrow & 4 < r < 5 \\ -\frac{70}{r^2} \longrightarrow & r > 5 \end{cases}$$

$\boxed{\text{Plot}}$

r	3^-	3^+	3.5	4^-	4^+	4.5	5^-	5^+
D_r	0	8	5.8	4.5	-7.5	-5.9	-4.8	-2.8

r	5.5	6
D_r	-2.3	-1.94



3 Maxwell's first eqⁿ :

Proof :

- $Q_{enc} = \int_v \rho_v \, dv$ (Cont. charge)
- $Q_{enc} = \oint_s \bar{D} \cdot d\bar{S}$ (Gauss law)
- $\oint \bar{D} \cdot d\bar{S} = \int_v \nabla \cdot \bar{D} \, dv$ (divergence Theorem)

From these relations

$$\int_v \rho_v \, dv = \int_v \nabla \cdot \bar{D} \, dv$$

$$\therefore \boxed{\rho_v = \nabla \cdot \bar{D}}$$

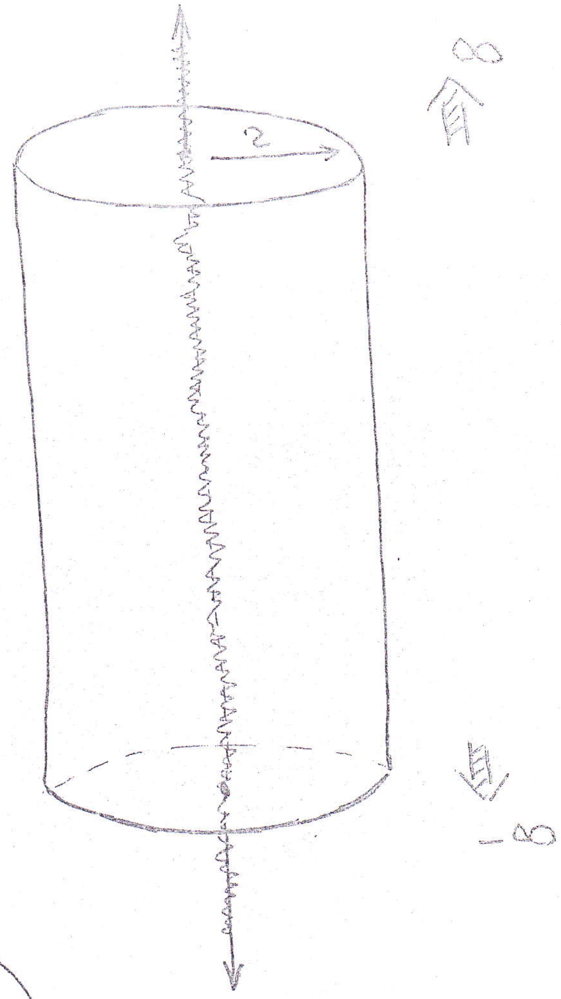
$\int_v \rho_v \, dv$ $\int_v \rho_v \, dv$

$\oint \bar{D} \cdot d\bar{S}$

$\oint \bar{D} \cdot d\bar{S} = \int \bar{\nabla} \cdot \bar{D} \, dv = \int_v \rho_v \, dv$

$\boxed{5}$ $\rho_L = 3\,\mu C/m$

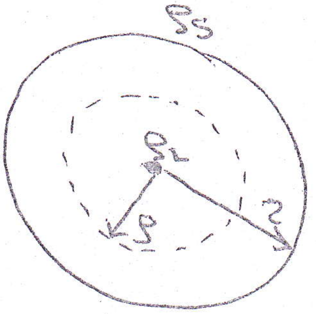
$$\rho_S = -\frac{1.5}{4\pi}\,\mu C/m^2$$



Using gauss law :

- gaussian sur : Cylinder with radius ρ .

(a) $0 < \rho < 2$:



(مسقط أفقي)

الشحنة المحصورة تكون الخط المشحون فقط

$Q_{enc} = \rho_L \cdot l$

$$\bar{D} \cdot \bar{S} = D_\rho \hat{\rho} \cdot \underbrace{2\pi\rho l}\,\hat{\rho}$$

سطح جاوس على شكل إسطوانة (نق = ρ)

$$\therefore\ 2\pi\rho \cancel{l} D_\rho = \rho_l \cdot \cancel{l}$$

$$\therefore\ D_\rho = \frac{\rho_L}{2\pi\rho}$$

$\bar{D} \cdot \bar{S} = D\rho\,\hat{\rho} \cdot 2\pi\rho l\,\hat{\rho}$

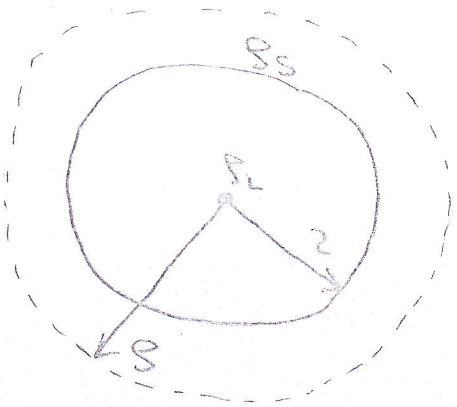
$D\rho = \frac{\rho_L}{2\pi\rho}$

(b) $\rho > 2$:

$Q_{enc} = \rho_\ell \cdot l + \rho_S \cdot S$

$= \rho_L \cdot l + \rho_S \cdot (2\pi(2)l)$

$= [\rho_L + \rho_S(4\pi)]\, l$



$\bar{D} \cdot \bar{S} = 2\pi\rho\, l\, D_\rho$

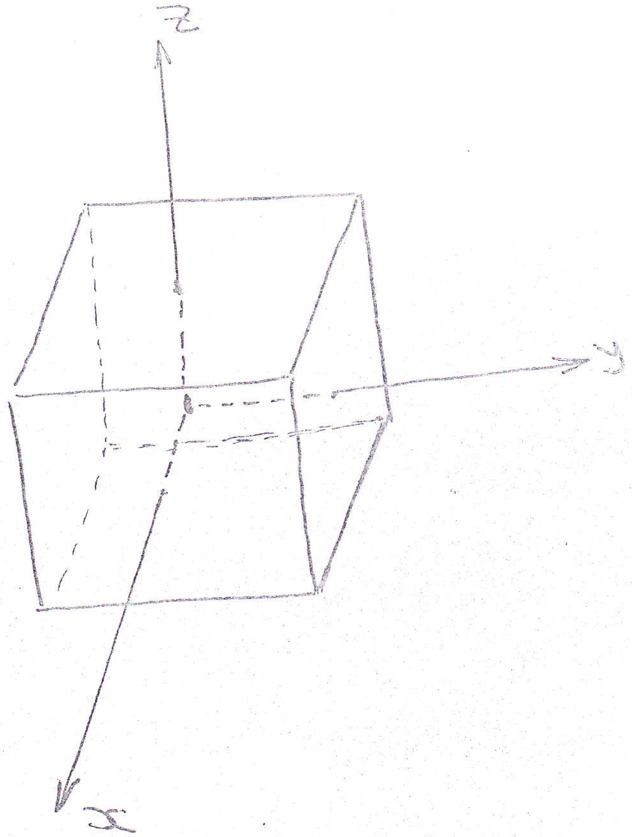
$\therefore\ 2\pi\rho\,\cancel{l}\,D_\rho = [\rho_L + 4\pi\rho_S]\,\cancel{l}$

$\therefore\ D_\rho = \frac{1}{2\pi\rho}[\rho_L + 4\pi\rho_S]$

$$\therefore \bar{D} = \begin{cases} \frac{0.4775}{\rho}\ \mu C/m^2 & 0 < \rho < 2 \\ \frac{0.2387}{\rho}\ \mu C/m^2 & \rho > 2 \end{cases}$$

$\bar{D} = 8x^2y^2\hat{x} + 4x^2y\,\hat{y}$

$\psi = \int \bar{D} \cdot d\bar{S}$



إذا أردنا حساب الـ ψ بهذه الطريقة

فإنه يوجد لدينا ستة أسطح مختلفة

$x = +0.5 \quad d\bar{S} = dy\,dz\,\hat{x}$

$x = -0.5 \quad d\bar{S} = -dy\,dz\,\hat{x}$

$y = +0.5 \quad d\bar{S} = dx\,dz\,\hat{y}$

$y = -0.5 \quad d\bar{S} = -dx\,dz\,\hat{y}$

$z = +0.5 \quad d\bar{S} = dx\,dy\,\hat{z}$

$z = -0.5 \quad d\bar{S} = -dx\,dy\,\hat{z}$

ولكن بكل بساطة يمكننا إستخدام :

__Maxwell First eqn :__

$$\rho_v = \nabla \cdot \bar{D}$$

in Cart : $\rho_v = \nabla \cdot \bar{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$

$\therefore \rho_v = 16xy^2 + 4x^2$

Then, we can use gauss law:

$$\Psi = Q_{enc}$$

$$Q_{enc} = \int_V \rho_v \, dv$$

$$= \iiint_{-0.5}^{0.5} (16x y^2 + 4x^2) \, dx \, dy \, dz$$

$$= \iint_{-0.5}^{0.5} \left[8x^2 y^2 + \frac{4}{3} x^3 \right] \Big|_{-0.5}^{0.5} \cdot dy \, dz$$

$$= \iint_{-0.5}^{0.5} [\, 8 \underset{\overset{\downarrow}{0}}{(0.5^2 - 0.5^2)} y^2 + \frac{4}{3} (0.5^3 + 0.5^3) \, dy \, dz$$

$$= \iint_{-0.5}^{0.5} \frac{1}{3} \, dy \, dz$$

$$= \frac{1}{3} \cdot 1 \cdot 1 = \frac{1}{3} C$$

$$\therefore \boxed{\Psi = Q_{enc} = \frac{1}{3} C}$$

$\boxed{7}$ $\bar{D} = \frac{20}{\rho}\cos\left(\frac{\phi}{2}\right)\hat{\phi}$

region: $1 \leq \rho \leq 2$, $0 \leq \phi \leq \pi$, $0 \leq z \leq 2.5$

Methode 1:

$20\rho^{-1}\cos\left(\frac{\phi}{2}\right)\hat{\phi}$

$-\frac{20}{\rho^2}\cos\left(\frac{\phi}{2}\right)\hat{\phi}$

Maxwell first eqn: $\rho_v = \nabla \cdot \bar{D}$

$$\nabla \cdot \bar{D} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}(h_2 h_3 D_{u1}) + \frac{\partial}{\partial u_2}(h_1 h_3 D_{u2}) + \frac{\partial}{\partial u_3}(h_1 h_2 D_{u3})\right]$$

for cyl: $h_1 = h_3 = 1$, $h_2 = \rho$

$u_1 = \rho$, $u_2 = \phi$, $u_3 = z$

$$\therefore \rho_v = \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{\partial}{\partial \phi}(D_\phi) + \frac{\partial}{\partial z}(\rho D_z)\right]$$

$$= \frac{1}{\rho}\left[\frac{\partial}{\partial \phi}\left(\frac{20}{\rho}\cos\frac{\phi}{2}\right)\right]$$

$$= -\frac{20}{\rho^2} \cdot \sin\left(\frac{\phi}{2}\right) \cdot \frac{1}{2}$$

$$\therefore \rho_v = -\frac{10}{\rho^2}\sin\left(\frac{\phi}{2}\right)$$

$$Q_{enc} = \int_V \rho_v \, dv \quad \Rightarrow \quad dv = \rho \, d\rho \, d\Phi \, dz$$

$$\therefore Q_{enc} = \int_0^{2.5}\int_0^{\pi}\int_1^{2} -\frac{10}{\rho^2} \sin\left(\frac{\Phi}{2}\right) \rho \, d\rho \, d\Phi \, dz$$

$$= -10 \ln\rho \Big|_1^2 \cdot \frac{\cos\left(\frac{\Phi}{2}\right)}{1/2} \Big|_{\pi}^{0} \cdot z \Big|_0^{2.5}$$

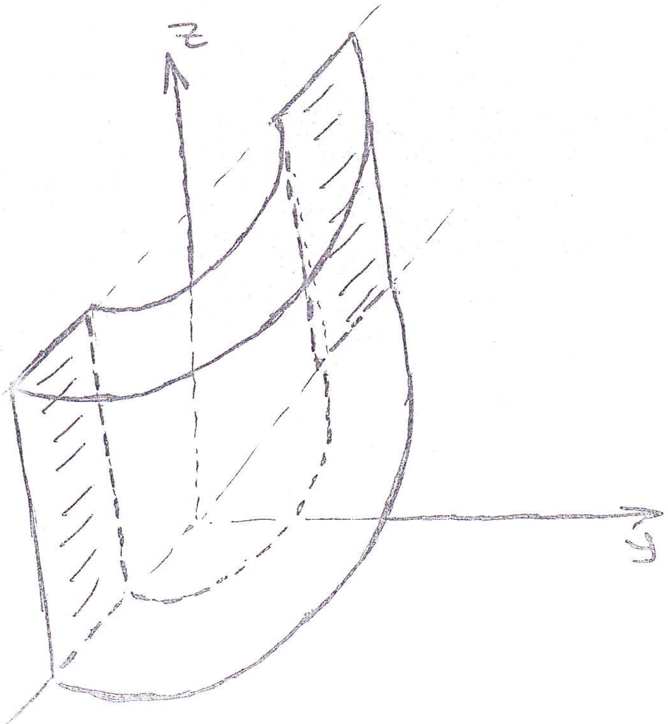
$$= -20 \left[\ln(2) - 0\right] \cdot \left[1 - 0\right] \cdot 2.5$$

$$= -50 \ln(2) \ C$$

$$\therefore \boxed{Q_{enc} = -50 \ln(2) \ C}$$

Methode 2 : Using gauss law :

$$Q_{enc} = \oint_S \bar{D} \cdot d\bar{S}$$



لدينا في هذا الرسم ستة أسطح

$\rho = 1$ (السطح الملفوف الداخلي)

$\rho = 2$ (السطح الملفوف الخارجي)

$\Phi = 0$ (السطح الأيسر)

$\Phi = \pi$ (السطح الأيمن)

السفلي

$z = 0$ (السطح الطولي)

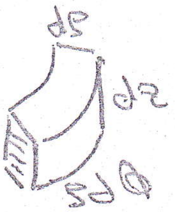
$\bar{D} = \frac{20}{\rho} \cos\left(\frac{\phi}{2}\right) \hat{\phi}$ $\Rightarrow$ in the $\hat{\phi}$ direction

$\therefore$ الأسطح التي نأخذها في الإعتبار هي الأسطح العمودية على الإتجاه $\hat{\phi}$ وهم :

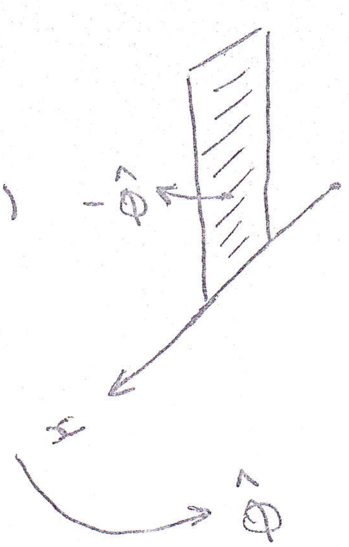
$\phi = 0$ and $\phi = \pi$

" الأسطح المؤشرة على الرسم "

$\therefore$ Sur $\phi = 0$:



(الفيض يخرج من الشكل)



$$\therefore d\bar{S} = d\rho\, dz\, (-\hat{\phi})$$
$$= -d\rho\, dz\, \hat{\phi}$$

$$\bar{D} = \frac{20}{\rho} \cos\left(\frac{\phi}{2}\right) \hat{\phi}$$

$$\therefore \bar{D} \cdot d\bar{S} = -\frac{20}{\rho} \cos\frac{\phi}{2}\, d\rho\, dz \quad (\text{sur } \phi = 0)$$
$$= -\frac{20}{\rho}\, d\rho\, dz$$

$$\therefore Q_{encl} = \int_0^{2.5} \int_1^2 -\frac{20}{\rho}\, d\rho\, dz$$
$$= -20 \ln(\rho)\Big|_1^2 \cdot z\Big|_0^{2.5}$$

$$= -50 \ln(2)$$

$\boxed{8}$ $\bar{D} = 4\rho^2 \, \hat{\rho} \; C/m^2 \dashrightarrow (\rho < 5)$

(a) from Maxwell's First eqn :

$$\rho_v = \nabla \cdot \bar{D} = \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{\partial}{\partial \phi}(D_\phi) + \frac{\partial}{\partial z}(\rho D_z)\right]$$

$$\rho_v = \frac{1}{\rho} \cdot \frac{\partial}{\partial \rho}(\rho D_\rho)$$

$$= \frac{1}{\rho} \cdot \frac{\partial}{\partial \rho}(4\rho^3) = \frac{1}{\rho}\left[12\rho^2\right]$$

$$\therefore \boxed{\rho_v = 12\rho}$$

at $\rho = 2m$: $\rho_v = 24 \; C/m^3$

(b)

⇛ way 1 :

$$\psi = Q_{enc} = \int_V \rho_v \, dV$$

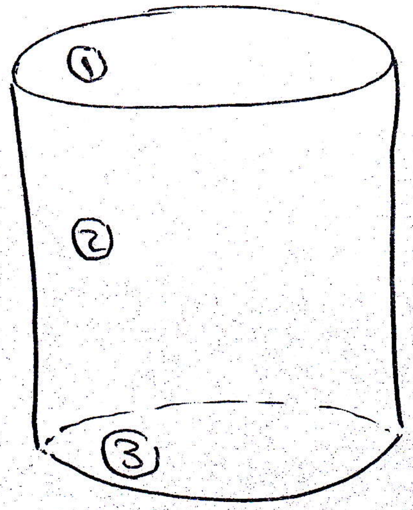
$$= \int_{-5}^{5}\int_{0}^{2\pi}\int_{0}^{2} 12\rho \; \rho d\rho d\phi dz$$

$$= 12 \cdot \frac{\rho^3}{3} \cdot \phi\Big|_0^{2\pi} \cdot z\Big|_{-5}^{5}$$

$$\therefore \boxed{\psi = 640\pi C}$$

⇛ Way (2):

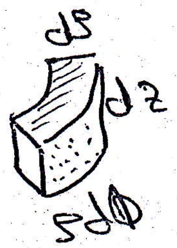
يوجد لدينا ثلاثة أسطح :



① $z=5$ $d\bar{S} = \rho\, d\rho\, d\phi\, \hat{z}$

② $\rho=2$ $d\bar{S} = \rho\, d\phi\, dz\, \hat{\rho}$

③ $z=-5$ $d\bar{S} = -\rho\, d\rho\, d\phi\, \hat{z}$



as $\bar{D} = 4\rho^2 \hat{\rho}$ ⇛ in $\hat{\rho}$ - dir.

∴ we only concern with the sur. $\rho = 2$

لأنه هو السطح الحوى على $\bar{D}$.

$$\therefore \bar{D} \cdot d\bar{S} = 4\rho^2 \hat{\rho} \cdot \rho\, d\phi\, dz\, \hat{\rho}$$

$$= 4\rho^3\, d\phi\, dz$$

$$\therefore \psi = \int_{-5}^{5} \int_{0}^{2\pi} 4\rho^3\, d\phi\, dz \quad \text{but } \boxed{\rho = 2}$$

$$= 4 * 2^3 * \phi \Big|_0^{2\pi} * z \Big|_{-5}^{5}$$

$$= 640\pi \ C$$

③ from Gauss law :

$Q_{enc} = \Psi = 640\pi C$

$\therefore Q_{enc} = 640\pi C.$

Mansoura University
Faculty of Engineering
Communication Engineering Department

Second Year Communication
Electromagnetic Fields
Sheet (3)

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[1] A point charge, $Q = 30$ nc, is located at the origin. Find the electric flux density **D** at (1, 3, -4) m.

[2] What net flux crosses the closed surface S shown in Fig.1 which contains a charge distribution in the form of a plane disk of radius 4m with a density $\rho_s = (\sin^2\varphi)/2\rho$ c/m^2.

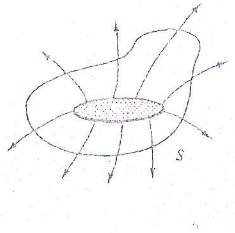

Fig. 1

[3] A uniform line charge of 15 nC/m lies along the z axis and a uniform sheet of charge of 4 nC/m^2 is located at the plane $z = 1$.

(a) What is the total electric flux leaving the sphere $r = 2$?

(b) What is the magnitude of the electric vector **D** at the point on the sphere where $x = 2$?

[4] The spherical surfaces r = 3, 4, and 5 m contain uniform surface charge densities of 8, - 12, and ρ_s nC/m^2, respectively. What value must ρ_s be so that $\mathbf{D} = 0$ for $r > 5$? If $\rho_s = 2$ nC/m^2; calculate and plot D_r vs. r for $0 \leq r \leq 6$ m.

[5] A uniform line charge of $\rho_l = 3$ μc/m lies along z axis, and a cylinder of radius 2m has $\rho_s = -1.5/4\pi$ μC/m^2. Both distributions are infinite in extent with z. Find **D** in all regions.

[6] Find the flux of the vector $\mathbf{D} = 8x^2y^2\,a_x + 4x^2y\,a_y$ leaving the surface of the cube: $-0.5 \leq x \leq 0.5$, $-0.5 \leq y \leq 0.5$, $-0.5 \leq z \leq 0.5$.

[7] If $\mathbf{D} = [(20\cos(\varphi/2))/\rho]\,a_\varphi$ in the region $1 \leq \rho \leq 2$, $0 \leq \varphi \leq \pi$, $0 \leq z \leq 2.5$ determine the total charge lying within the given region.

[8] Within the cylindrical region $\rho \leq 5$ m, the electric flux density is given as $4\rho^2\,a_\rho$ C/m^2.

(a) What is the volume charge density at $\rho = 2$m?

(b) How much electric flux leaves the cylinder $\rho = 2$, $-5 \leq z \leq 5$?

(c) How much charge is contained within this cylinder?